Society of Land Surveyors of Iowa

Errors:
Measurement to Product

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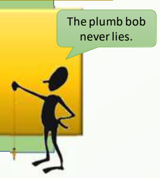


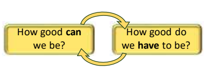
Regardless the equipment used, the personnel involved, survey measurements will include errors.

Almost every survey operation at some point will require measurements. That means the operation results will, in part, depend on the measurement errors.

How much error is acceptable? It depends.

Depending on the operation, the error level is either up to the surveyor or dictated by some rule or law.

Either way requires understanding where errors come from, how they behave, how they can be controlled, and determining their effect on our final decisions.



Society of Land Surveyors of Iowa

Subjects

I. Measurement Errors
II. Random Errors
III. Least Squares
IV. Standards and Specifications

I. Measurement Errors

A. Quality
B. Error Tenets
C. Sources
D. Types
E. Minimizing

A. Quality

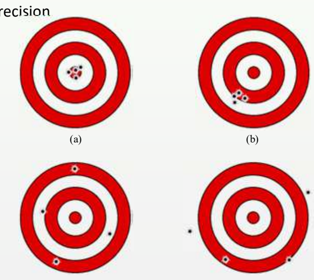
Accuracy
- Absolute nearness to true value; measurement with minimal errors.

Precision
- Repeatability; spread of a measurement set.
- Similar error behavior in repeated measurements

Resolution
- The smallest division to which the measurement can be expressed based on the instrumentation used.

A. Quality

Accuracy and Precision

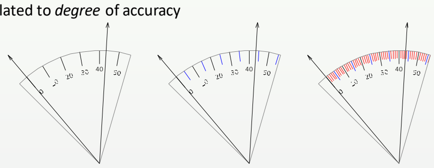


I. Measurement Errors

A. Quality

Resolution

Related to *degree* of accuracy



direct: 10°
interpolate: 1°

direct: 5°
interpolate: 1°

direct: 1°
interpolate: 0.5°

I. Measurement Errors

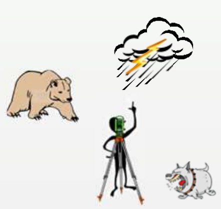
B. Error Tenets

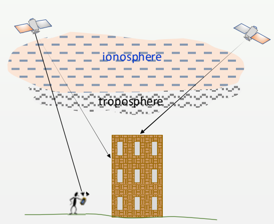
No measurement is exact.
Every measurement contains errors.
The true value of a measurement is never known.
The exact error present in a measurement is unknown.

I. Measurement Errors

C. Error Sources - Where they come from

1. Natural
   Environment within which a measurement is made

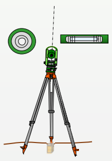


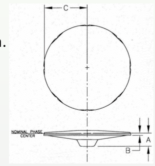


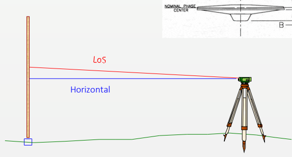
I. Measurement Errors

C. Error Sources - Where they come from

2. Instrumental
   Measuring equipment condition; calibration.



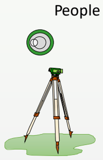


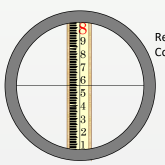


I. Measurement Errors

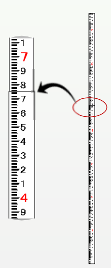
C. Error Sources - Where they come from

3. Personal
   People making the measurement





Record 8.55.
Common newbie mistake.



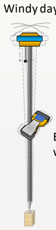
I. Measurement Errors

D. Error Types - How they behave

1. Mistake
   Carelessness or misunderstanding

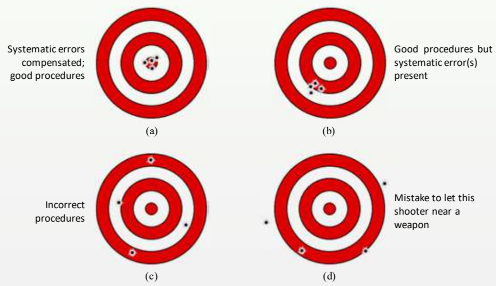
2. Systematic
   Conform to mathematical or physical law

3. Random
   Remaining error after mistakes & systematic errors are eliminated
   Tend to be small; as likely positive as negative.



I. Measurement Errors

D. Error Types - How they behave



I. Measurement Errors

E. Minimizing

1. Mistake

Generally, with a single measurement, can't tell if a mistake was made.

Remeasure until all measurements are within an acceptable tolerance

Tolerance can be defined as:

- A specific spread, eg, angles must all be within 10"
- A precision level, eg, distances must be within 1/5000
- Different based on equipment or personnel, eg, angles within 10" for experienced operator, 25" for new tech.

Reject measurements outside the tolerance to avoid biasing good measurements.

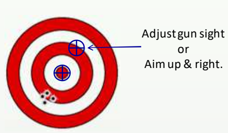
I. Measurement Errors

E. Minimizing

2. Systematic

Apply a correction:

- Mathematic
- Mechanical
- Procedure



Specific surveying procedures are used to:

- ensure measurement is made correctly
- allow some systematic errors to cancel

I. Measurement Errors

E. Minimizing

2. Systematic

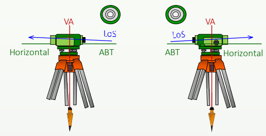
Example: Level collimation error

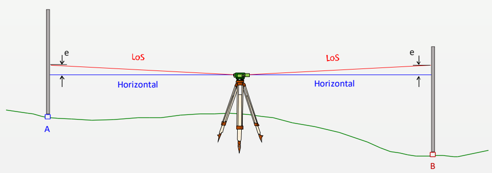
Balance BS and FS distances

Two-peg test

Adjust

Math corr'n





I. Measurement Errors

E. Minimizing

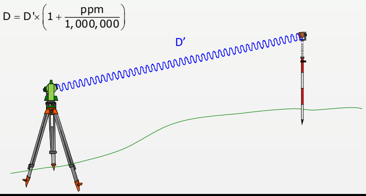
2. Systematic

Example: Atmospheric conditions electronic distance

Math correction, ppm

$$ppm = 278.96 - \frac{10.5 \times P}{1 + 0.002175 \times T}$$

$$D = D' \times \left(1 + \frac{ppm}{1,000,000}\right)$$



I. Measurement Errors

E. Minimizing

3. Random

Appropriate equipment

Knowledgeable personnel

Favorable conditions

Repeat measurements

How many times?

Example: FGCS Triangulation angle measurement specs

Order/Class	1st	2nd / I	2nd / II	3rd / I	3rd / II
Num Positions	16	16	8 or 12*	4	2
Max Std Dev of Mean	0.4″	0.5″	0.8″	1.2″	2.0″
Reject Limit from Mean	4″	4″	5″	5″	5″

a position is 1 D/R measurement

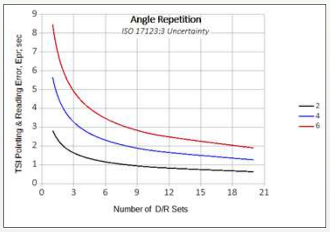
I. Measurement Errors

E. Minimizing

3. Random

Comparison of different resolution Total Stations

Pointing and reading error



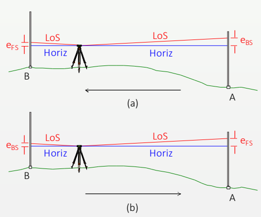
I. Measurement Errors

E. Minimizing

3. Random

A measurement must be independent.

Not changing conditions allows mistakes and systematic errors to be repeated.



Leveling

From a single set up:

(a) BS A and FS B

(b) BS B and FS A

Collimation error does not compensate.

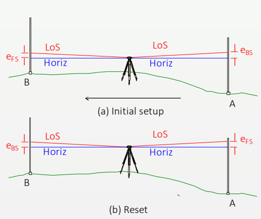
I. Measurement Errors

E. Minimizing

3. Random

A measurement must be independent.

Change conditions (reset inst, resight tagets, diff day, combo, etc).



Leveling

As a minimum, reset level between runs:

(a) Balance BS & FS dists;
BS A then FS B

(b) Reset with BS/FS dists balanced;
BS B then FS A

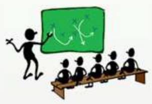
I. Measurement Errors

E. Minimizing

3. Random

To minimize errors:

- Favorable conditions
- Use proper equipment
- Trained personnel
- Correct procedures
- Repeat measurements



This will minimize but not eliminate errors.

Only random errors will be left and they should be small.

I. Measurement Errors

Quiz:

Source	Type	Error
N I P	M S R	Mis-coding an RTK point
N I P	M S R	Centering total station over a point
N I P	M S R	Earth Curvature
N I P	M S R	Atmospheric refraction
N I P	M S R	Multipath
N I P	M S R	GPS Position accuracy [eg, ±(5 mm + 1 ppm)
N I P	M S R	Auto level sticking compensator

I. Measurement Errors

Quiz:

Source	Type	Error
N I **P**	**M** S R	Mis-coding an RTK point
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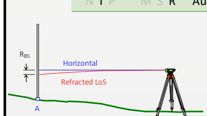
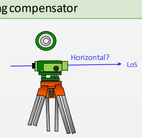
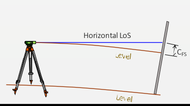
N I P	M S **R**	Multipath
N **I** P	M S **R**	GPS Position accuracy [eg, ±(5 mm + 1 ppm)
N **I** P	M S **R**	Auto level sticking compensator







II. Random Errors

A. Basic Analysis
B. Error Propagation
C. Summary

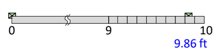
II. Random Errors

A. Basic Analysis

1. Terms

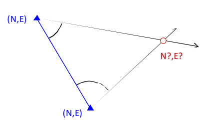
Direct measurement

Measure the unknown with instrumentation



Indirect measurement

Determine the unknown from measurements and computations



Want coordinates, but can't measure them directly.
Perform an angle-angle intersection.

II. Random Errors

A. Basic Analysis

1. Terms

Redundancy

Also known as a degree of freedom (df)
A measurement beyond what is needed to determine a quantity.

Example: a horizontal distance is measured once
- 1 measurement
- 1 unknown – the distance
- 0 redundancy

Measure again
- 2 measurements
- 1 unknown
- 1 redundancy

etc.

II. Random Errors

A. Basic Analysis

1. Terms

Discrepancy

Difference between any two measurements in a set

Most Probable Value, MPV

The most likely value of the unknown based on the measurement set.
If all measurements are equal quality, then the MPV is the arithmetic average.

$$M = \frac{\sum m_i}{n}$$

m: measurement
n: number of measurements

II. Random Errors

A. Basic Analysis

1. Terms

Residual, v

The difference between the MPV and a measurement.

$$v_i = M - m_i$$

Least Squares

The MPV is the value which minimizes the sum of the squared residuals.

$$\sum (v_i^2) = \min$$

Least sum of the *Squares*.
Any other number will increase the sum.

This is based on statistical behavior of random errors.

II. Random Errors

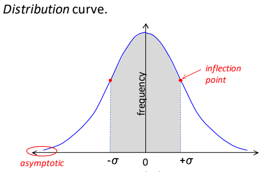
A. Basic Analysis

1. Terms

Standard Deviation, σ

The *Normal Distribution* (aka, *Bell*) curve is a graph of residuals frequency.
Area under the curve is the total measurement probability
σ is approx. 68% of the area under the *Normal Distribution* curve.
Precision indicator of a measurement set.

$$\sigma = \sqrt{\frac{\sum (v_i^2)}{(n-1)}}$$



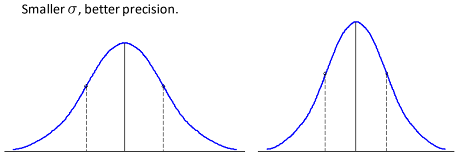
II. Random Errors

A. Basic Analysis

1. Terms

Standard Deviation, σ

The smaller the σ, the less dispersed the measurements.
Smaller σ, better precision.



Measurement set 1
Larger σ

Measurement set 2
Smaller σ, better precision

II. Random Errors

A. Basic Analysis

1. Terms

Confidence Interval, CI

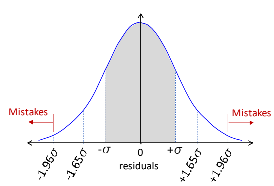
Certainty that a measurement will fall within a range.

$\pm\sigma$ is a 68% CI

Other common CI are:

CI	*Value*	
90%	≈ 1.65(σ)	
95%	≈ 1.96(σ)	aka *2 sigma**
100%?		

*95% CI is often used to find mistakes.



II. Random Errors

A. Basic Analysis

1. Terms

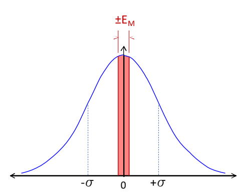
Standard Error of the Mean, E_M

σ is for the measurement set

E_M is the expected error in the MPV

An expected accuracy indicator.

$$E_M = \frac{\sigma}{\sqrt{n}}$$



II. Random Errors

A. Basic Analysis

2. Example

Measurement num	m	$v = M\text{-}m_i$	v^2
1	45.66		
2	45.68		
3	45.66		
4	45.65		
sums:	*182.65*		

$df = 4 - 1 = 3$

II. Random Errors

A. Basic Analysis

2. Example

Measurement num	m	$v = M\text{-}m_i$	v^2
1	45.66	+0.002	0.000004
2	45.68	-0.018	0.000324
3	45.66	+0.002	0.000004
4	45.65	+0.012	0.000144
sums:	*182.65*		*0.000476*

$df = 4 - 1 = 3$

$$M = \frac{182.65}{4} = 45.6625 = 45.662$$

$$\sigma = \sqrt{\frac{\sum v_i^2}{n-1}} = \sqrt{\frac{0.000476}{4-1}} = \pm 0.0125963 = \pm 0.012$$

$$E_M = \frac{\sigma}{\sqrt{n}} = \frac{\pm 0.0125963}{\sqrt{4}} = \pm 0.006298 = \pm 0.006$$

II. Random Errors

A. Basic Analysis

3. Comparisons

Different measurement sets can be compared for precision and accuracy.

Example: Two crew measured different angles, multiple times.

	Crew A	Crew B
Num of meas	2 D/R	4 D/R
Average angle	128°18’15”	196°02’40”
Std Dev	±0°00’12”	±0°00’14”

1 D/R = 2 angle meas.

Which crew has:

Better precision?

Better expected accuracy?

II. Random Errors

A. Basic Analysis

3. Comparisons

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Std Dev	±0°00′12″	±0°00′14″

1 D/R = 2 angle meas.

Which crew has:

Better precision?

Crew A because it has a smaller standard deviation

Better expected accuracy?

Crew A: $E_M = \frac{\pm 12''}{\sqrt{2\times 2}} = \pm 06.0''$

Crew B: $E_M = \frac{\pm 14''}{\sqrt{4\times 2}} = \pm 04.9''$

II. Random Errors

A. Basic Analysis

4. Weights

Different quality measurements can be used together.

Use relative *weights* to give better quality measurements greater effect.

Weights can be based on

- standard deviations
- equipment specs
- equipment setup errors
- number of setups
- number of repetitions
- guesstimates (experience)

$$M_w = \frac{\sum (w_i \times m_i)}{\sum w_i}$$

$$\sigma_w = \sqrt{\frac{\sum (w_i v_i^2)}{(n-1)\left(\sum w_i\right)}}$$

II. Random Errors

A. Basic Analysis

4. Weights

Example: A distance is measured by different methods with these results.

Method	Dist, m	σ
Steel Tape	254.63	±0.21
Subtense bar	254.69	±0.16
Total Station	254.72	±0.06
sums:	*764.04*	

Unit mean: $M = \frac{764.04}{3} = 254.680$

Unit means singular or one.

Treating all the measurements the same is like using a unit weight = 1

II. Random Errors

A. Basic Analysis

4. Weights

Example: A distance is measured by different methods with these results.

Method	Dist, m	σ	$w=1/\sigma^2$	w x m
Steel Tape	254.63	±0.21	22.7	5780.1
Subtense bar	254.69	±0.16	39.1	9958.4
Total Station	254.72	±0.06	277.8	70,761.2
sums:	*764.04*		*339.6*	*86,499.7*

Unit mean: $M = \frac{764.04}{3} = 254.680$

Weighted mean: $M_w = \frac{\sum (w_i \times m_i)}{\sum w_i} = \frac{86499.7}{339.6} = 254.711$

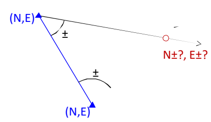
Mean is pulled toward the Total Station distance

II. Random Errors

B. Error Propagation

1. Indirect measurements

Uncertainty in desired quantity is a function of the measurement uncertainties.

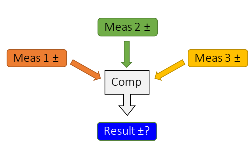


II. Random Errors

B. Error Propagation

1. Indirect measurements

The measurements are combined in some mathematical operation.
How the error propagates depends on the mathematical operation.



Because measurement errors are random, they do not propagate in a simple fashion by adding or multiplying.

II. Random Errors

B. Error Propagation

1. Indirect measurements

There are as many ways to propagate errors as there are equations to combine them.

Two common ones used in Surveying are:

a. Error of a Sum

Adding, subtracting measurements

b. Error of a Series

When the same error recurs multiple times

II. Random Errors

B. Error Propagation

2. Error of a Sum

$$E_{Sum} = \sqrt{E_1^2 + E_2^2 + \cdots + E_n^2}$$

E_1, E_2, ..., E_n are the uncertainties in the numbers being added/subtracted.

Example:

A soil specimen is divided into three sample parts which are separately weighed. What is the error in the soil's total weight?

Soil sample	Weight
1	56.2 gr ±0.15 gr
2	28.9 gr ±0.21 gr
3	41.6 gr ±0.11 gr
	127.6 gr

$$E_{Sum} = \sqrt{(\pm 0.15)^2 + (\pm 0.21)^2 + (\pm 0.11)^2}$$
$$= \pm 0.2815 = \underline{\pm 0.28}$$

II. Random Errors

B. Error Propagation

3. Error of a Series

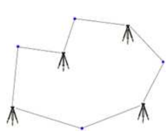
$$E_{Series} = E\sqrt{n}$$

E is an expected error each time an operation is performed.
Total error is individual expected error times the square root of the chances it has to occur.

Example:

Each time they set up a level, a survey crew can determine an elevation difference to ±0.015 ft. What is their error on this loop?

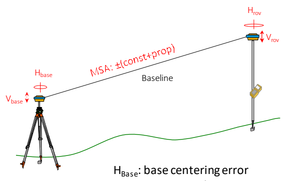
$$E_{Sum} = \pm 0.015\sqrt{4} = \underline{\pm 0.030}$$



II. Random Errors

B. Error Propagation

Example: GPS horizontal and vertical baseline components



Positioning performance

Precision Static	H: 3 mm + 0.1 ppm V: 3.5 mm + 0.4 ppm
Static/Fast Static[1]	H: 3 mm + 0.5 ppm V: 5 mm + 0.8 ppm
PPP	H: 3 cm RMS[2] V: 5 cm RMS[2] Convergence time: < 5 mins[3]
RTK[4]	H: 5 mm + 0.5 ppm V: 10 mm + 0.8 ppm
RTK, TILT Compensated	RTK + 5 mm + 0.5 mm / ° tilt Compensation up to 60°

H_{Base}: base centering error
H_{Rov}: rover centering error

V_{Base}: base HI error
V_{Rov}: rover HI error

$$E_{Sum} = \sqrt{E_1^2 + E_2^2 + \cdots + E_n^2}$$

$$E_H = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Rov}^2}$$

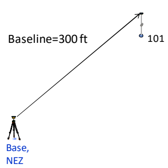
$$E_V = \sqrt{V_{Base}^2 + V_{MSA}^2 + V_{Rov}^2}$$

II. Random Errors

B. Error Propagation

Example: GPS horizontal and vertical baseline components

MSA: ±(5mm + 0.5ppm) horiz
Base centering error: E_B=±0.005ft
Rover centering error: E_R= ±0.05ft
What is baseline length error?



$$E_H = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Rov}^2}$$

$$5mm \times \frac{39.37in}{1000mm} \times \frac{1ft}{12in} = 0.0164ft$$

$$E_H = \sqrt{(0.005ft)^2 + \left(0.0164ft + \frac{300ft \times 0.5}{1,000,000}\right)^2 + (0.05ft)^2}$$

$$= \underline{\pm 0.053ft}$$

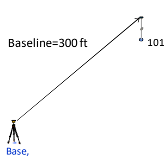
$$precision: \frac{0.053ft}{300.00ft} = \frac{1}{5360}$$

II. Random Errors

B. Error Propagation

Example: GPS horizontal and vertical baseline components

MSA: ±(10mm + 0.8ppm) vert
Base HI error: E_B=±0.005ft
Rover HI error: E_R= ±0.005ft
What is baseline elev diff error?



$$E_V = \sqrt{V_{Base}^2 + V_{MSA}^2 + V_{Rov}^2}$$

$$10mm \times \frac{39.37in}{1000mm} \times \frac{1ft}{12in} = 0.03281ft$$

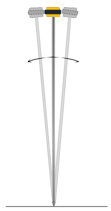
$$E_V = \sqrt{(0.005ft)^2 + \left(0.03281ft + \frac{300ft \times 0.8}{1,000,000}\right)^2 + (0.005ft)^2}$$

$$= \underline{\pm 0.034ft}$$

II. Random Errors

B. Error Propagation

Example: What about tilt compensation?



Positioning performance

Precision Static	H: 3 mm + 0.1 ppm V: 3.5 mm + 0.4 ppm
Static/Fast Static[1]	H: 3 mm + 0.5 ppm V: 5 mm + 0.8 ppm
PPP	H: 3 cm RMS[2] V: 5 cm RMS[2] Convergence time: < 5 mins[3]
RTK[4]	H: 5 mm + 0.5 ppm V: 10 mm + 0.8 ppm
RTK, TILT Compensated	RTK + 5 mm + 0.5 mm / ° tilt Compensation up to 60°

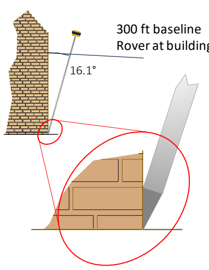
$$E_{H} = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Tilt}^2}$$

II. Random Errors

B. Error Propagation

Example: What about tilt compensation?

300 ft baseline
Rover at building base



MSA: ±(5mm + 0.5ppm) horiz
Tilt: RTK + 5mm + 0.5mm/°
Base centering error: E_B=±0.005ft
What is baseline length error?

$$5mm = 0.0164ft$$

$$16.1° \times 0.5mm/° = 8.05mm \times \frac{39.37in}{1000mm} \times \frac{1ft}{12in} = 0.02641ft$$

$$E_{H} = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Tilt}^2}$$

$$= \sqrt{0.005ft^2 + \left(0.0164ft + \frac{300ft \times 0.5}{1,000,000}\right)^2 + (0.0164ft + 0.02641)^2}$$

$$= \pm 0.046ft$$

$$Prec = \frac{0.046ft}{300.00ft} = \frac{1}{6520}$$

II. Random Errors

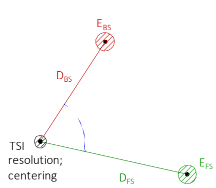
B. Error Propagation

4. Combined

Angle measurement

Angles are complicated because there are 3 points and a number of random error sources:

- Instrument angle resolution
- Instrument centering
- Target centering at BS and FS points
- Target sightings
- Distances to BS and FS points
- Number of times the angle is measured.



II. Random Errors

B. Error Propagation

4. Combined

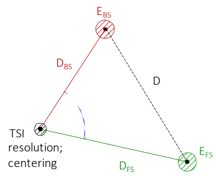
Angle measurement

TSI Point & Read $E_{pr} = \frac{2 \times E_{DIN}}{\sqrt{n}}$

TSI Centering $E_{tsi} = \frac{D \times E_i}{D_{BS} D_{FS} \sqrt{2}} \times \frac{206,264.8 \text{ sec}}{\text{radian}}$

Target Centering $E_t = \sqrt{\left(\frac{E_{BS}}{D_{BS}}\right)^2 + \left(\frac{E_{FS}}{D_{FS}}\right)^2} \times \frac{206,264.8 \text{ sec}}{\text{radian}}$

Angle Error $E_{ang} = \sqrt{E_{pr}^2 + E_{tsi}^2 + E_t^2}$



II. Random Errors

B. Error Propagation

4. Combined

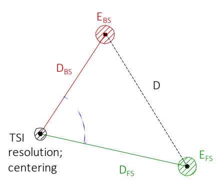
Example

A TSI with DIN spec of 2 sec;
centering error: ±0.005 ft

BS & FS centering errors: ±0.005 ft & ±0.01 ft;
BS & FS dists: 176 ft & 243 ft.

Angle was measured 2 D/R to get 123°30'10"

Many comps later....
Expected error in the angle is ±11.4"

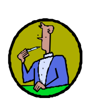


II. Random Errors

C. Summary

Error Management

- Errors can't be eliminated but effects can be minimized
  - Systematic errors
    - Procedure
    - Adjustment
    - Computation
  - Random Errors
    - Appropriate equipment
    - Knowledgeable personnel
    - Careful measuring techniques
  - Remaining random errors
    - Individual measurements - Analyze
    - Networks - Adjust and analyze



II. Random Errors

Now what?

Random error will exist and must be dealt with.

- Minimize by repetition.
- Adjustment: Distribute part of the error back into each measurement or result.

Adjustment should reflect measurement error behavior

- Results might not fit perfectly
- Appropriate weights
- Apply a "best fit" adjustment model

III